RANDOM VIBRATION—AN OVERVIEW

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ABSTRACT

Random vibration is becoming increasingly recognized as the most realistic method of simulating the dynamic environment of military applications. Whereas the use of random vibration specifications was previously limited to particular missile applications, its use has been extended to areas in which sinusoidal vibration has historically predominated, including propeller driven aircraft and even moderate shipboard environments. These changes have evolved from the growing awareness that random motion is the rule, rather than the exception, and from advances in electronics which improve our ability to measure and duplicate complex dynamic environments.

The purpose of this article is to present some fundamental concepts of random vibration which should be understood when designing a structure or an isolation system.

INTRODUCTION

Random vibration is somewhat of a misnomer. If the generally accepted meaning of the term "random" were applicable, it would not be possible to analyze a system subjected to "random" vibration. Furthermore, if this term were considered in the context of having no specific pattern (i.e., haphazard), it would not be possible to define a vibration environment, for the environment would vary in a totally unpredictable manner.

Fortunately, this is not the case. The majority of random processes fall in a special category termed stationary. This means that the parameters by which random vibration is characterized do not change significantly when analyzed statistically over a given period of time - the RMS amplitude is constant with time. For instance, the vibration generated by a particular event, say, a missile launch, will be statistically similar whether the event is measured today or six months from today. By implication, this also means that the vibration would be statistically similar for all missiles of the same design. It is possible to subdivide a process into a number of sub-processes, each of which could be considered to be stationary. For example, a missile environment could consist of several stationary processes, such as: captive carry, buffet, launch and free flight. Each of these sub-processes have unique amplitude, frequency and time characteristics, requiring separate analyses and considerations.

The assumption of a stationary process is essential in both a technical and legal sense. As previously stated, it would not be possible for a designer to analyze a system, nor for a user to test a system prior to installation in the field, if the vibration excitation were not stationary. Consequently, it would not be possible to develop a legally binding specification. In subsequent conversations, it is assumed that the random excitation is a stationary process.

Any vibration is described by the time history of motion, where the amplitude of the motion is expressed in terms of displacement, velocity or acceleration. Sinusoidal vibration is the simplest motion, and can be fully described by straightforward mathematical equations. Figure 1 shows the amplitude time plot of a sinusoidal vibration, and indicates that sinusoidal vibration is cyclic and repetitive. In other words, if frequency and amplitude (or time and amplitude) are defined, the motion can be predicted at any point in time.

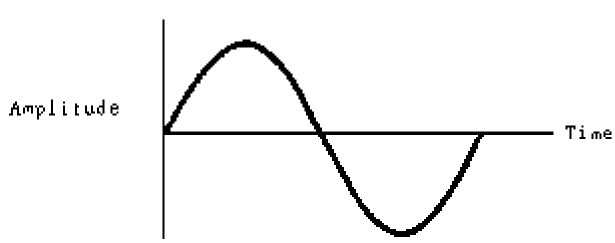


Figure 1. Amplitude-Time History of Sinusoidal Vibration

A random vibration is one whose absolute value is not predictable at any point in time. As opposed to sinusoidal vibration, there is no well defined periodicity - the amplitude at any point in time is not related to that at any other point in time. Figure 2 shows the amplitude time history of a random vibration. The lack of periodicity is apparent. A major difference between sinusoidal vibration and random vibration lies in the fact that for the latter, numerous frequencies may be excited at the same time. Thus structural resonances of different components can be excited simultaneously, the interaction of which could be vastly different from sinusoidal vibration, wherein each resonance would be excited separately.

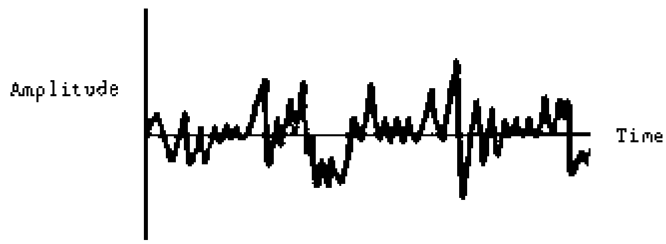


Figure 2. Amplitude-Time History of Random Vibration

Although the instantaneous amplitude of a random vibration cannot be expressed mathematically as an exact function of time, it is possible to determine the probability of occurrence of a particular amplitude on a statistical basis.

To characterize a stationary process, an ensemble of possible time histories must be obtained, wherein the amplitude is measured over the frequency range of excitation. Thus, the three parameters of interest are: frequency, time and amplitude. This information would provide the ability to analyze a random process in a statistical sense. The characterization of random vibration typically results in a frequency spectrum of Power Spectral Density (PSD) or Acceleration Spectral Density (ASD), which designates the mean square value of some magnitude passed by a filter, divided by the bandwidth of the filter. Thus, Power Spectral Desnity defines the distribution of power over the frequency range of excitation.

The equipment designer is interested in avoiding mechanical failure and equipment malfunction. These may be produced by different mechanisms, requiring different methods of corrective action. To the designer, random vibration could be considered as either:

a) an infinite number of harmonic vibrations with unpredictable amplitude and phase relationships in the frequency domain; or

b) the sum of an infinite number of infinitesimal shocks occurring randomly in the time domain.

In the first case, response at a particular frequency may be the primary concern. For example, when a displacement sensitive device is excited at its natural frequency, relatively large displacements may result in malfunction. In such a case, the malfunction might be corrected by reducing the amplitude of excitation at the particular frequency of concern - the natural frequency of the device. This might be accomplished by inserting a vibration isolator between the source of excitation and the device. Alternatively, displacement might be reduced by adjusting the stiffness of the device, or by increasing damping at the natural frequency of the device.

If the random vibration is considered as an infinite number of infinitesimal shocks, the overall Grms may result in a fatigue related structural failure of a component due to the intermittent shocks associated with the random excitation. In this case, the problem might be corrected by reducing the overall Grms or by increasing the strength of the component.

There is a relationship between these considerations. The nature of the equipment problem will affect the type of corrective action to be investigated.

STATISTICAL ASPECTS

Statistics is the science of predicting the probability of occurrence of a particular event. In random vibration, it may be desired to predict the probability of a response exceeding a particular value. For instance, if a black box has a ¼-inch clearance to an adjacent structure, it is ncessary to know the probability of the black box impacting the structure. Alternatively, since the probability of impact would never be absolutely zero, it would also be of interest to predict the average time between successive impacts, or the average number of times a particular amplitude may occur in a given duration. This would be of interest in calculating acceleration (or stress), and relating these values to the fatigue life of a component.

The most commonly used probability distribution is the Normal (Gaussian) distribution. The probability density function for a normal distribution is given by:

$$P = \frac{1}{\sqrt{2\pi}} e^{[(-1/2)(X/Xrms)^2]}$$

Equation 1

Equation 1 is plotted as Figure 3, which is the probability of occurrence of the ratio of the instantaneous value to the RMS value. In Equation 1, X and Xrms could have units of displacement, velocity or acceleration, or derivatives of these terms.

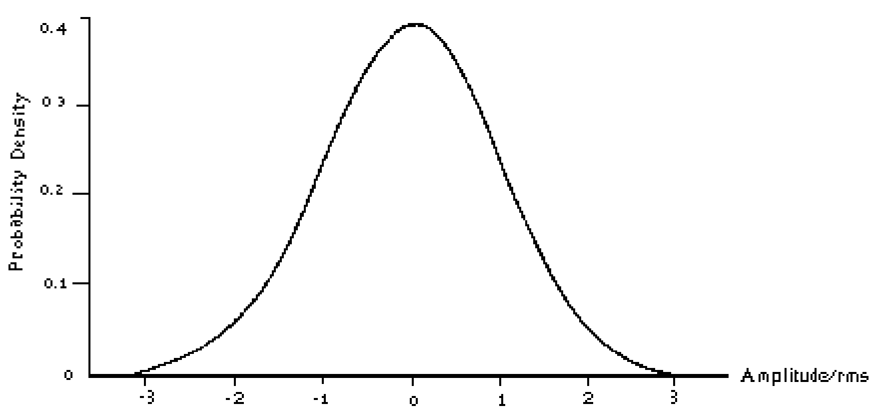


Figure 3. Probability Density Function for a Normal Distribution

The probability that an amplitude lies between two values is equal to the area under the normal curve between the two values. By definition, the total area under the curve is equal to unity. Thus, if the event in question is absolutely certain to happen, the area under the curve, the probability of occurrence, is equal to 1.0. If the event is certain not to happen, the area under the curve, the probability of occurrence, is zero. Probabilities are positive numbers between zero and one, and can be expressed as a percentage. For example, the probability that the magnitude of X/Xrms falls between +1 and -1 is equal to the shaded area shown on Figure 3. This area is .68, which means that there is a 68% probability that the actual amplitude will be between the X/Xrms value of +1 and -1. This is commonly referred to as the "one sigma" probability. The corresponding two sigma and three sigma probabilities are .95 and .997, respectively.

Table I gives areas under the normal distribution curve for various values of X/Xrms.

Since the probability density function approaches zero assymptotically, the probability of a particular event not occurring will never be exactly zero. Such being the case, what is a reasonable approach in evaluating random vibration? Returning to the problem of the black box with a ¼-inch clearance to an adjacent structure, if Xrms were .0833 inches, the three sigma value would be .25 inches (3 x.0833), and the probability of impact would be .3%. The probability of impact could be reduced by reducing Xrms. For example, if Xrms were decreased to .0625, a four sigma deflection would be required to cause impact, and the probability of impact would be reduced to .001%.

The generally accepted procedure is to use a three sigma value for design purposes. In some cases, the use of higher values may be justified. Frequently, there are additional restraints precluding the use of higher values.

CHARACTERIZATION OF INPUT ACCELERATION AND DISPLACEMENT

The simplest random excitation to analyze is a band limited white spectrum shown in Figure 4.

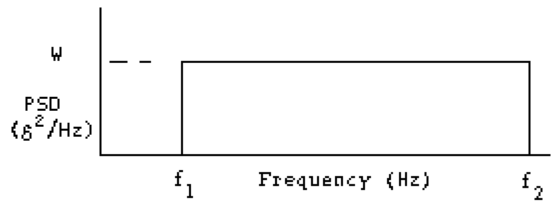


Figure 4. Band Limited White Spectrum

The overall input Grms is the square root of the area under the curve, i.e.,

$$G_{rms} = \sqrt{W\ (f_2 - f_1)} \qquad \text{Equation 2}$$

This value could be used in Equation 1 to predict the probability of occurrence of instantaneous values of acceleration for a random signal. For design purposes, Grms would generally be multiplied by three to provide three sigma values.

In actual practice, it is not possible for a vibration shaker to generate the instantaneous drop off shown in Figure 4. Consequently, specifications generally show a roll off rate in terms of decibels per octave (dB/Oct.). Figure 5 shows a possible input vibration spectra including roll-off rates.

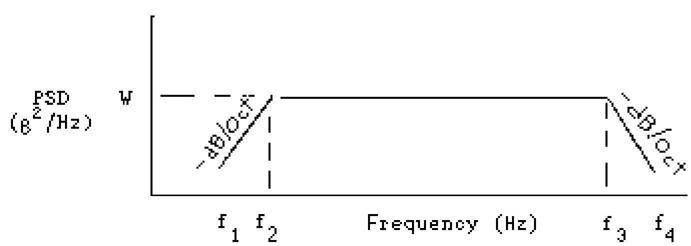


Figure 5. Possible Random Vibration Input Sectrum

As in Figure 4, the overall Grms is the square root of the total area under the curve. However, since most curves are plotted as straight lines on log-log paper, calculating the area under the sloped lines is more complicated than for the region of constant PSD. Appendix A provides equations used to calculate G^2 for sloped lines. To determine Grms for a spectrum with numerous break points, G^2. for all areas are summed, and the square root of this summation results in overall Grms.

A decibel is a logarithmic notation for expressing ratios between two quantities. For Power Spectral Density (g^2/Hz), the following equation is used to relate two values of PSD:

$$\Delta dB = 10 \log [W_1/W_2] \qquad \text{Equation 3}$$

Figure 5 could be constructed using this equation, and knowing that an octave is a doubling or halving of frequency (e.g., 8 Hz and 16 Hz are separated by one octave as are 80 Hz and 160 Hz).

Displacement could be analyzed in the same manner as acceleration, except that rather than using units of g^2/Hz, the units would be in^2/Hz. The RMS displacement would be the square root of the area under the curve of in^2/Hz. However, since accelerometers are the most frequently used method of measuring random vibration, alternate methods are used to determine displacement. For a band limited white spectrum, the RMS displacement can be shown to be given by:

$$X_{rms} = G_{rms} \times \frac{g}{4\pi^2}\sqrt{\frac{1}{3}\quad\frac{f_2^{\,3} - f_1^{\,3}}{\left(f_1^{\,2}\right)\ \left(f_2^{\,3}\right)}\quad\frac{1}{f_2 - f_1}} \qquad \text{Equation 4}$$

where, G_{rms} = input acceleration

g = acceleration constant

= 386 in./sec.2

f_1 = lower frequency, Hz

f_2 = upper frequency, Hz

For most cases, f_2 is significantly higher than f_l and Equation 4 can be approximated by:

$$X_{rms} = \frac{565\, G_{rms}}{\sqrt{f_1^3\, f_2}} \qquad \text{Equation 5}$$

Equation 5 represents the one sigma stroke capability necessary to generate the power spectral density having an overall acceleration, Grms, and upper and lower frequencies f_2 and f_l, respectively. Equations 4 and 5 could be used in conjunction with Equation 1 to determine the probability of occurrence of a particular input displacement. In practice, input displacement is typically clipped electronically at the three sigma value, as is the three sigma input Grms.

ISOLATION OF RANDOM VIBRATION

If a mass supported on vibration isolators is subjected to a stationary, Gaussian random vibration input, the response will be a stationary, Gaussian random response. Insertion of vibration isolators will change the amplitudes of response, such as Power Spectral Density and Grms, but the previously discussed statistical concepts remain applicable.

The Power Spectral Densities of the isolated equipment (response) are related to those at the foundation (input) by the following equation

$$W_{out} = W_{in}\, T^2_A \qquad \text{Equation 6}$$

where, W_{out} = Power Spectral Density on isolated equipment, g^2/Hz

W_{in} = Input Power Spectral Density, g^2/Hz

T_A = Absolute Transmissibility of Isolation System

Thus, insertion of vibration isolators modifies the frequency response to a random vibration input in a similar manner to a sinusoidal vibration input. As in sinusoidal vibration, there is a frequency region of amplification and a frequency region of attenuation. However, in random vibration, the amplitude of the response is modified by transmissibility squared, whereas, in sinusoidal vibration, the response is a linear function of transmissibility. This difference is due to the fact that in random vibration, we deal with power, whereas, in sinusoidal vibration, we deal with acceleration.

The RMS acceleration (Grms) transmitted to the isolated equipment is equivalent to the square root of the area under the curve of the response Power Spectral Density (g^2/Hz). Thus,

$$G_{rms} = \sqrt{\int W_{in}\; T_A^2\; df} \qquad \text{Equation 7}$$

For a band limited white spectrum, Equation 7 can be simplified to the following:

$$G_{rms} = \sqrt{\frac{\pi}{2} f_n \; T_A \; W_{in}}$$ Equation 8

where, f_n = Isolation System Natural Frequency, Hz

T_A = Absolute Tranmissibility

W_{in} = Input Power Spectral Density, g^2/Hz

Frequently, the relative displacement of a component is required in order to ensure that there is sufficient clearance to prevent metal-to-metal contact. If Grms transmitted to the isolated equipment is known, the RMS displacement is given by:

$$\delta_{rms} = 9.75\ G_{rms} / f_n^2$$ Equation 9

Equation 9 is useful since it provides the ability to calculate relative displacement based on two directly measureable parameters - G_{rms} and f_n.

If the random vibration excitation is a band limited white spectrum, the RMS displacement can be determined from:

$$\sqrt{\frac{W_{in} \; T_A}{f_n^3}}$$ Equation 10

The RMS displacement calculated by Equations 9 or 10 are typically multiplied by three to determine the minimum clearance required to prevent metal-to-metal contact.

As previously mentioned, the probability of particular event not occurring will never be absolutely zero. There are times when it may be necessary to estimate the average number of occurrences (N) of a particular event in a given time duration (t). For a band limited white spectrum, the average number of occurrences in a given time duration can be estimated by:

$$N = 2\ f_n\ \Delta t\ e^{[(-1/2)(X/Xrms)2\ ]}$$ Equation 11

where, f_n = Natural Frequency, Hz

Δt = Time Interval, Seconds

X/X_{rms} = Amplltude Ratio

To demonstrate, if the previously discussed black box has a ¼ inch clearance (X), and if Xrms (also δ rms) is .0833 inches, a three sigma deflection will cause metal-to-metal contact. If the isolation system natural frequency is 25 Hz, there would be an average of 33 impacts in a one minute period. If Xrms were reduced to .0625 inches, the average number of impacts in a one minute period would be reduced to one.

The mean time between consecutive occurrences may be a more meaningful statistic than the average number of occurrences within a given time interval. This can be determined by setting N=1 in Equation 11, and solving for Δt. Thus,

$$\Delta t = \frac{1}{2fn} e^{[(1/2)(x/Xrms)^2]}$$

Equation 12

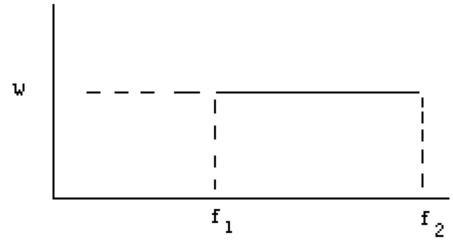
If the clearance between the black box and the adjacent structure were increased to .50 inches, and if Xrms were .0833 inches and fn equal to 25 Hz, the mean time between consecutive impacts would be approximately 370 hours.

APPENDIX A

Calculating G_{rms} for Spectra of Various Shapes

I. FLAT SPECTRUM

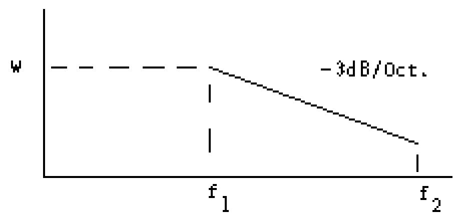
$$G^2 = W \times (f_2 - f_1)$$



II. WHEN SLOPE IS DEFINED IN TERMS OF dB/OCTAVE

1) For the unique case of a slope of -3dB/Octave:

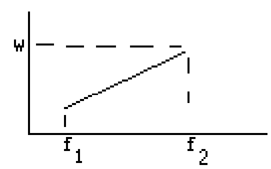
$G^2 = W \times f_1 \times \ln [f_2 / f_1]$



2) For positive slopes:

$Z_1 = (R / 3) + 1$, where R is slope in dB/Octave.

$$G^2 = \frac{W \times f_2}{Z_1} \quad 1 - \left(\frac{f_1}{f_2}\right)$$



3) for negative sopes:

$Z2 = (R / 3) -1$, where R is the absolute value of the slope in dB/Octave.

$$G^2 = \frac{W \times f_1}{Z_2} \quad 1 - \frac{1}{\left[\frac{f_2}{f_1}\right]^{Z_2}}$$

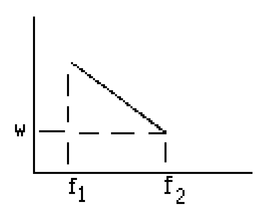
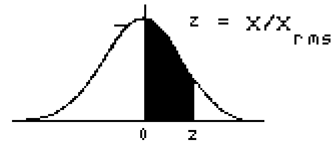


TABLE I

AREAS
under the
STANDARD
NORMAL CURVE
from 0 to z



z	0	1	2	3	4	5	6	7	8	9
0.0	.0000	.0040	.0040	.0120	.0160	.0199	.0239	.0279	.0319	.0359
0.1	.0398	.0438	.0478	.0517	.0557	.0596	.0636	.0675	.0714	.0754
0.2	.0793	.0832	.0871	.0910	.0948	.0987	.1026	.1064	.1103	.1141
0.3	.1179	.1217	.1255	.1293	.1331	.1368	.1406	.1442	.1480	.1517
0.4	.1554	.1591	.1628	.1664	.1700	.1736	.1772	.1808	.1844	.1879
0.5	.1915	.1950	.1985	.2019	.2054	.2088	.2123	.2157	.2190	.2224
0.6	.2258	.2291	.2324	.2357	.2389	.2422	.2454	.2486	.2518	.2549
0.7	.2580	.2612	.2642	.2673	.2704	.2734	.2764	.2794	.2823	.2852
0.8	.2881	.2910	.2939	.2967	.2996	.3023	.3051	.3078	.3106	.3133
0.9	.3159	.3186	.3212	.3238	.3264	.3289	.3315	.3340	.3365	.3389
1.0	.3413	.3438	.3461	.3485	.3508	.3531	.3554	.3577	.3599	.3621
1.1	.3643	.3665	.3686	.3708	.3729	.3749	.3770	.3790	.3810	.3830
1.2	.3849	.3869	.3888	.3907	.3925	.3944	.3962	.3980	.3997	.4015
1.3	.4032	.4049	.4066	.4082	.4099	.4115	.4131	.4147	.4162	.4177
1.4	.4192	.4207	.4222	.4236	.4251	.4265	.4279	.4292	.4306	.4319
1.5	.4332	.4345	.4357	.4370	.4382	.4394	.4406	.4418	.4429	.4441
1.6	.4452	.4463	.4474	.4484	.4495	.4505	.4515	.4525	.4535	.4545
1.7	.4554	.4564	.4573	.4582	.4591	.4599	.4608	.4616	.4625	.4633
1.8	.4641	.4649	.4656	.4664	.4671	.4678	.4686	.4693	.4699	.4706
1.9	.4713	.4719	.4726	.4732	.4738	.4744	.4750	.4756	.4761	.4767
2.0	.4772	.4778	.4783	.4788	.473	.4798	.4803	.4808	.4812	.4817
2.1	.4821	.4826	.4830	.4834	.4838	.4842	.4846	.4850	.4854	.4857
2.2	.4861	.4864	.4868	.4871	.4875	.4878	.4881	.4884	.4887	.4890
2.3	.4893	.4896	.4898	.4901	.4904	.4906	.4909	.4911	.4913	.4916
2.4	.4918	.4920	.4922	.4925	.4927	.4929	.4931	.4932	.4934	.4936
2.5	.4938	.4940	.4941	.4943	.4945	.4946	.4948	.4949	.4951	.4952
2.6	.4953	.4955	.4956	.4957	.4959	.4960	.4961	.4962	.4963	.4964
2.7	.4965	.4966	.4967	.4968	.4969	.4970	.4971	.4972	.4973	.4974
2.8	.4974	.4975	.4976	.4977	.4977	.4978	.4979	.4979	.4980	.4981
2.9	.4981	.4982	.4982	.4983	.4984	.4984	.4985	.4985	.4986	.4986
3.0	.4987	.4987	.4987	.4988	.4988	.4989	.4989	.4989	.4990	.4990
3.1	.4990	.4991	.4991	.4991	.1992	.4992	.4992	.4992	.4993	.4993
3.2	.4993	.4993	.4994	.4994	.4999	.4994	.4994	.4995	.4995	.4995
3.3	.4995	.4995	.4995	.4996	.4996	.4996	.4996	.4996	.4996	.4997
3.4	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4998
3.5	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998
3.6	.4998	.4998	.4999	.4999	.4999	.4999	.4999	.4999	.4999	.4999
3.7	.4999	.4999	.4999	.4999	.4999	.4999	.4999	.4999	.4999	.4999
3.8	.4999	.4999	.4999	.4999	.4999	.4999	.4999	.4999	.4999	.4999
3.9	.5000	.5000	.5000	.5000	.5000	.5000	.5000	.5000	.5000	.5000